Homework # 2 - SOLUTION

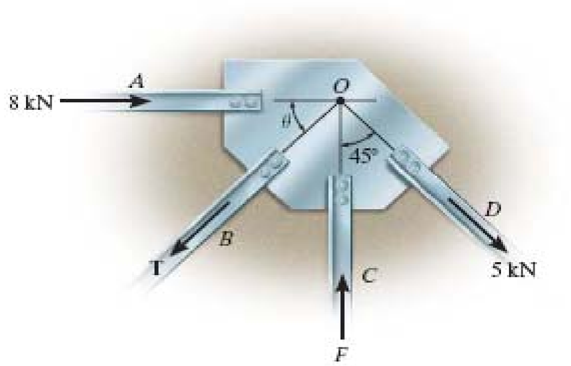
CIVE210 – STATICS

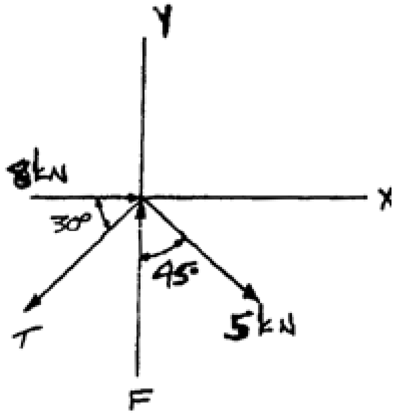
Topics: Equilibrium of a Particle (Chapter 3)

Textbook: Engineering Mechanics, by R.C. Hibbeler
Pearson, 12th Edition

Problems:
Chapter 3: Problems 3-5, 3-9, 3-14, 3-20, 3-22
Problems 3-30, 3-40, 3-42 (2-D Force System)
Problems 3-47, 3-56, 3-66 (3-D Force System)

3–5. The members of a truss are connected to the gusset plate. If the forces are concurrent at point *O*, determine the magnitudes of **F** and **T** for equilibrium. Take θ = 30°





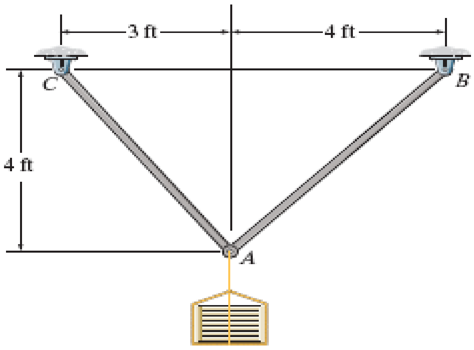
$\xrightarrow{+} \Sigma F_x = 0;$ $-T\cos 30° + 8 + 5\sin 45° = 0$

$T = 13.32 = 13.3$ kN **Ans**

$+\uparrow \Sigma F_y = 0;$ $F - 13.32\sin 30° - 5\cos 45° = 0$

$F = 10.2$ kN **Ans**

3–9. If members *AC* and *AB* can support a maximum tension of 300 lb and 250 lb, respectively, determine the largest weight of the crate that can be safely supported.



Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free - body diagram in Fig. (a),

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\cos 45° - F_{AC}\left(\frac{3}{5}\right) = 0 \qquad (1)$$

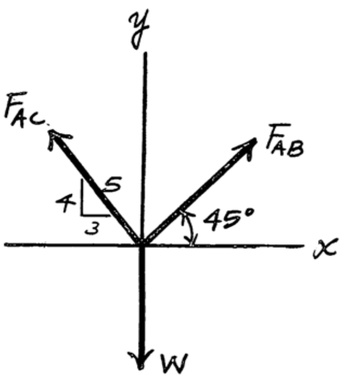
$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\sin 45° + F_{AC}\left(\frac{4}{5}\right) - W = 0 \qquad (2)$$

Assuming that rod *AB* will break first, $F_{AB} = 250$ lb. Substituting this value into Eqs. (1) and (2),

$F_{AC} = 294.63$ lb

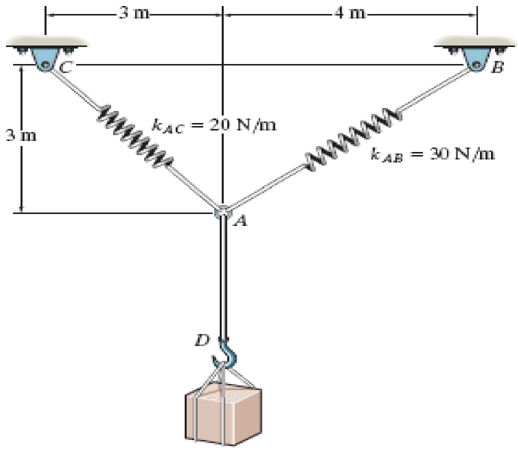
$W = 412$ lb **Ans.**

Since $F_{AC} = 294.63\text{ lb} < 300$ lb, rod *AC* will not break as assumed.



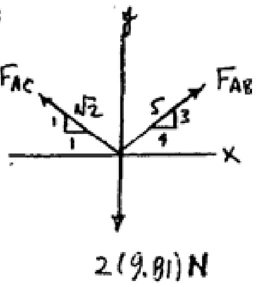
(a)

3–14. Determine the stretch in springs *AC* and *AB* for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.



$$F_{AD} = 2(9.81) = x_{AD}(40)$$

$$x_{AD} = 0.4905 \text{ m}$$ **Ans**



$$\xrightarrow{+}\Sigma F_x = 0; \qquad F_{AB}\left(\frac{4}{5}\right) - F_{AC}\left(\frac{1}{\sqrt{2}}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{1}{\sqrt{2}}\right) + F_{AB}\left(\frac{3}{5}\right) - 2(9.81) = 0$$

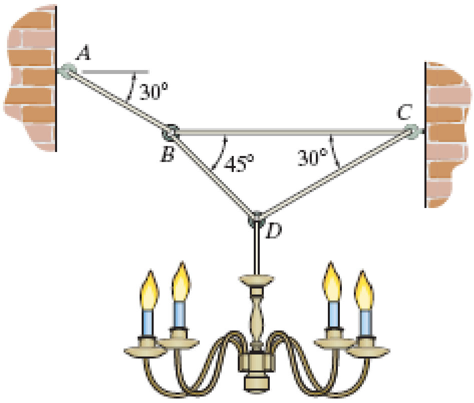
$$F_{AC} = 15.86 \text{ N}$$

$$x_{AC} = \frac{15.86}{20} = 0.793 \text{ m}$$ **Ans**

$$F_{AB} = 14.01 \text{ N}$$

$$x_{AB} = \frac{14.01}{30} = 0.467 \text{ m}$$ **Ans**

3–20. Determine the tension developed in each wire used to support the 50-kg chandelier.



Equations of Equilibrium: First, we will apply the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. (a).

$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $F_{CD}\cos 30° - F_{BD}\cos 45° = 0$ (1)

$+\uparrow\Sigma F_y = 0;$ $F_{CD}\sin 30° + F_{BD}\sin 45° - 50(9.81) = 0$ (2)

Solving Eqs. (1) and (2), yields

$F_{CD} = 359\text{ N}$ $F_{BD} = 439.77\text{ N} = 440\text{ N}$ **Ans.**

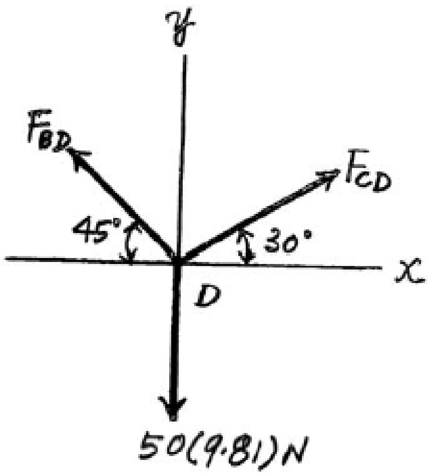
Using the result $F_{BD} = 439.77$ N and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint B shown in Fig. (b).

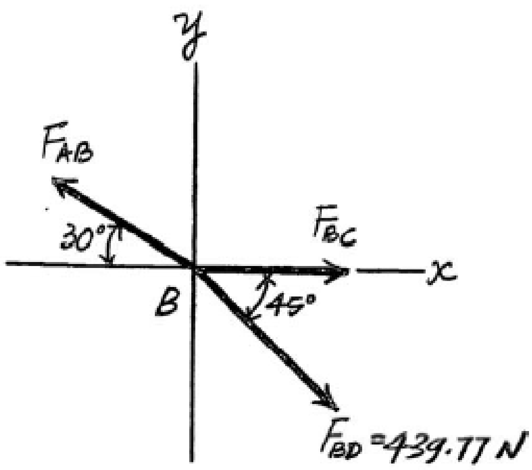
$+\uparrow\Sigma F_y = 0;$ $F_{AB}\sin 30° - 439.77\sin 45° = 0$

$F_{AB} = 621.93\text{N} = 622\text{ N}$ **Ans.**

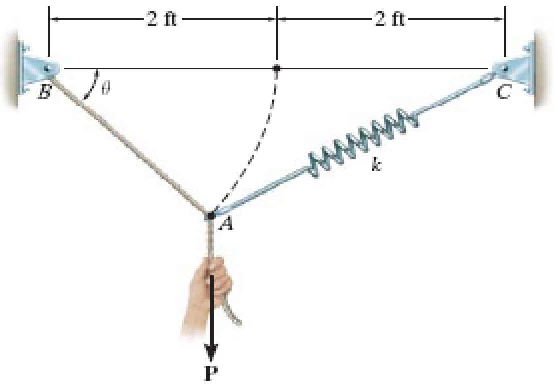
$\overset{+}{\rightarrow}\Sigma F_x = 0;$ $F_{BC} + 439.77\cos 45° - 621.93\cos 30° = 0$

$F_{BC} = 228\text{ N}$ **Ans.**



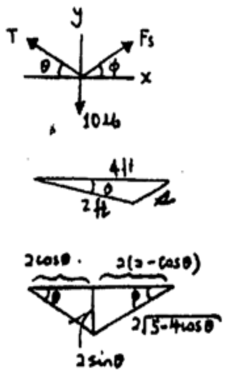


3–22. A vertical force **P** = 10 lb is applied to the ends of the 2-ft cord *AB* and spring *AC*. If the spring has an un-stretched length of 2 ft, determine the angle θ for equilibrium. Take k = 15 lb/ft.



$\xrightarrow{+}\Sigma F_x = 0;\quad F_s\cos\phi - T\cos\theta = 0 \quad (1)$

$+\uparrow \Sigma F_y = 0;\quad T\sin\theta + F_s\sin\phi - 10 = 0 \quad (2)$



$s = \sqrt{(4)^2 + (2)^2 - 2(4)(2)\cos\theta} - 2 = 2\sqrt{5-4\cos\theta} - 2$

$F_s = ks = 2k(\sqrt{5-4\cos\theta} - 1)$

From Eq. (1): $T = F_s\left(\frac{\cos\phi}{\cos\theta}\right)$

$T = 2k\left(\sqrt{5-4\cos\theta} - 1\right)\left(\frac{2-\cos\theta}{\sqrt{5-4\cos\theta}}\right)\left(\frac{1}{\cos\theta}\right)$

From Eq. (2):

$$\frac{2k\left(\sqrt{5-4\cos\theta} - 1\right)(2-\cos\theta)}{\sqrt{5-4\cos\theta}}\tan\theta + \frac{2k\left(\sqrt{5-4\cos\theta} - 1\right)2\sin\theta}{2\sqrt{5-4\cos\theta}} = 10$$

$$\frac{\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}}(2\tan\theta - \sin\theta + \sin\theta) = \frac{10}{2k}$$

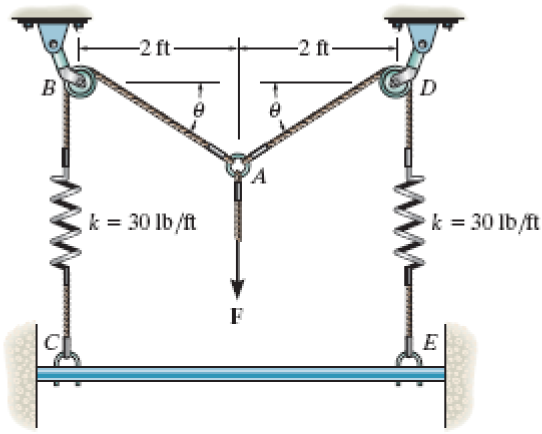
$$\frac{\tan\theta\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}} = \frac{10}{4k}$$

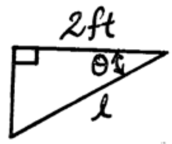
Set $k = 15$ lb/ft

Solving for θ by trial and error,

$\theta = 35.0°$ **Ans**

3–30. The springs on the rope assembly are originally un-stretched when $\theta = 0$. Determine the tension in each rope when **F** = 90 lb. Neglect the size of the pulleys at *B* and *D*.





$$l = \frac{2}{\cos\theta}$$

$$T = kx = k(l - l_0) = 30\left(\frac{2}{\cos\theta} - 2\right) = 60\left(\frac{1}{\cos\theta} - 1\right) \qquad (1)$$

$+\uparrow \Sigma F_y = 0; \quad 2T\sin\theta - 90 = 0 \qquad (2)$

Substituting Eq.(1) into (2) yields :

$120(\tan\theta - \sin\theta) - 90 = 0$

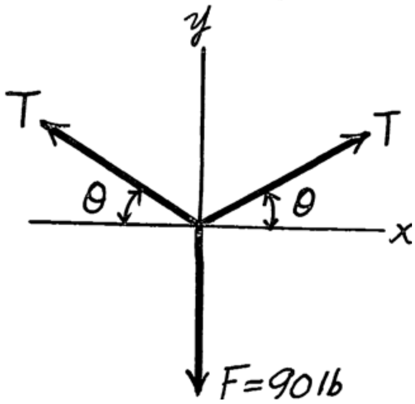
$\tan\theta - \sin\theta = 0.75$

By trial and error :

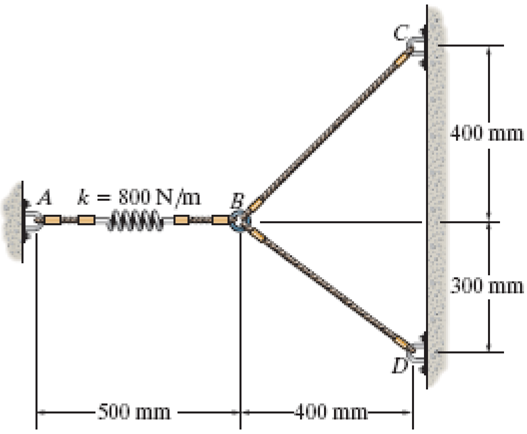
$\theta = 57.957°$

From Eq.(1),

$$T = 60\left(\frac{1}{\cos 57.957°} - 1\right) = 53.1 \text{ lb} \qquad \text{Ans}$$



3–40. The spring has a stiffness of k = 800 N/m and an un-stretched length of 200 mm. Determine the force in cables *BC* and *BD* when the spring is held in the position shown.



The Force in The Spring : The spring stretches $s = l - l_0 = 0.5 - 0.2 = 0.3$ m. Applying Eq.3 – 2, we have

$$F_{sp} = ks = 800(0.3) = 240 \text{ N}$$

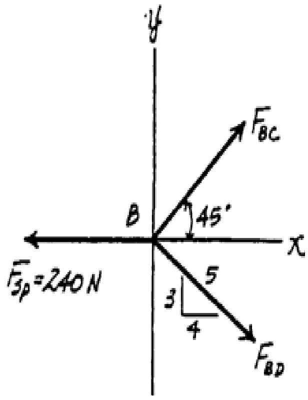
Equations of Equilibrium :

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC}\cos 45° + F_{BD}\left(\frac{4}{5}\right) - 240 = 0$$

$$0.7071F_{BC} + 0.8F_{BD} = 240 \qquad [1]$$

$$+\uparrow \Sigma F_y = 0; \quad F_{BC}\sin 45° - F_{BD}\left(\frac{3}{5}\right) = 0$$

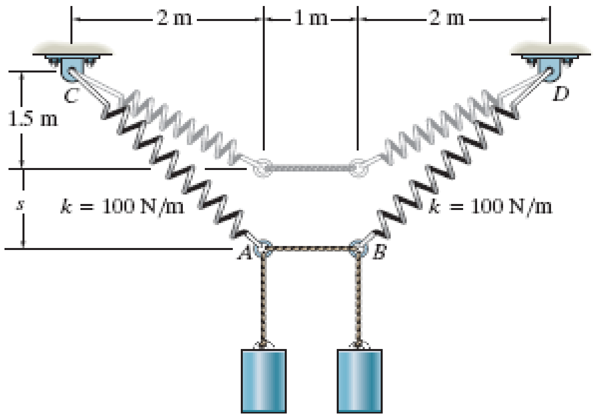
$$F_{BC} = 0.8485F_{BD} \qquad [2]$$

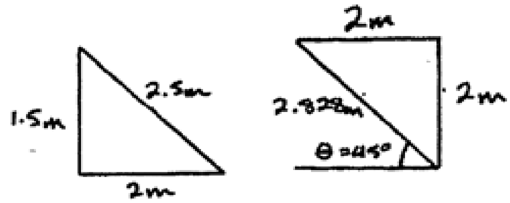


Solving Eqs.[1] and [2] yields,

$F_{BD} = 171$ N $\quad F_{BC} = 145$ N **Ans**

3–42. Determine the mass of each of the two cylinders if they cause a sag of s = 0.5 m when suspended from the springs at *A* and *B*. Note that s =0 when the cylinders are removed.

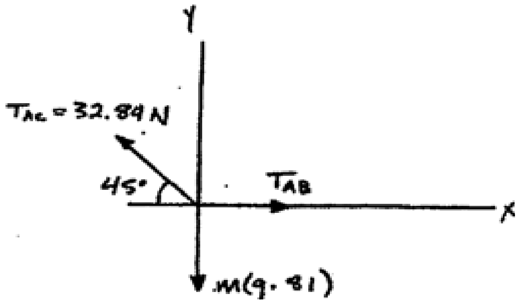




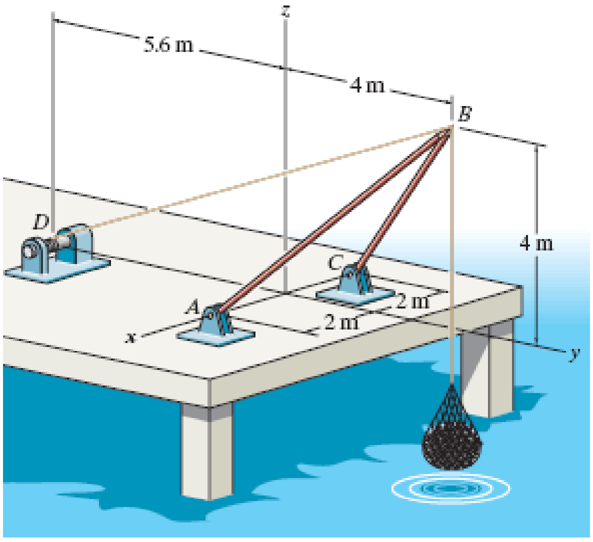
$$T_{AC} = 100\ \text{N/m}\,(2.828 - 2.5) = 32.84\ \text{N}$$

$+\uparrow \Sigma F_y = 0;\qquad 32.84 \sin 45° - m(9.81) = 0$

$m = 2.37\ \text{kg}$ **Ans**



3–47. The shear leg derrick is used to haul the 200-kg net of fish onto the dock. Determine the compressive force along each of the legs *AB* and *CB* and the tension in the winch cable *DB*. Assume the force in each leg acts along its axis.



$$\mathbf{F}_{AB} = F_{AB}\left(-\frac{2}{6}\mathbf{i} + \frac{4}{6}\mathbf{j} + \frac{4}{6}\mathbf{k}\right)$$

$$= -0.3333\,F_{AB}\,\mathbf{i} + 0.6667\,F_{AB}\,\mathbf{j} + 0.6667\,F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{CB} = F_{CB}\left(\frac{2}{6}\mathbf{i} + \frac{4}{6}\mathbf{j} + \frac{4}{6}\mathbf{k}\right)$$

$$= 0.3333\,F_{CB}\,\mathbf{i} + 0.6667\,F_{CB}\,\mathbf{j} + 0.6667\,F_{CB}\,\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}\left(-\frac{9.6}{10.4}\mathbf{j} - \frac{4}{10.4}\mathbf{k}\right)$$

$$= -0.9231\,F_{BD}\,\mathbf{j} - 0.3846\,F_{BD}\,\mathbf{k}$$

$$\mathbf{W} = -1962\,\mathbf{k}$$

$$\Sigma F_x = 0; \qquad -0.3333\,F_{AB} + 0.3333\,F_{CB} = 0$$

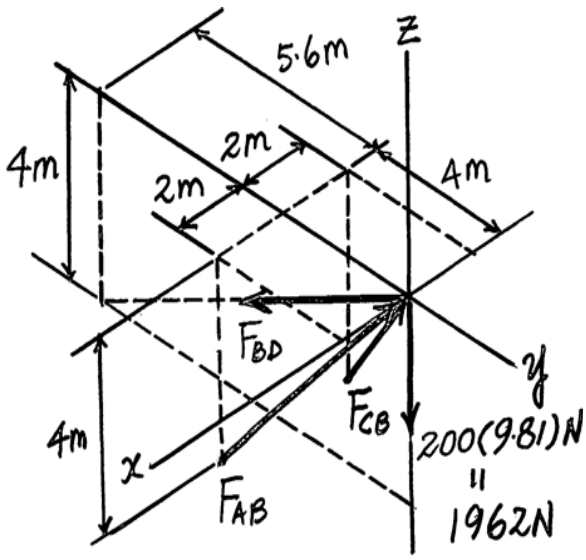
$$\Sigma F_y = 0; \qquad 0.6667\,F_{AB} + 0.6667\,F_{CB} - 0.9231\,F_{BD} = 0$$

$$\Sigma F_z = 0; \qquad 0.6667\,F_{AB} + 0.6667\,F_{CB} - 0.3846\,F_{BD} - 1962 = 0$$

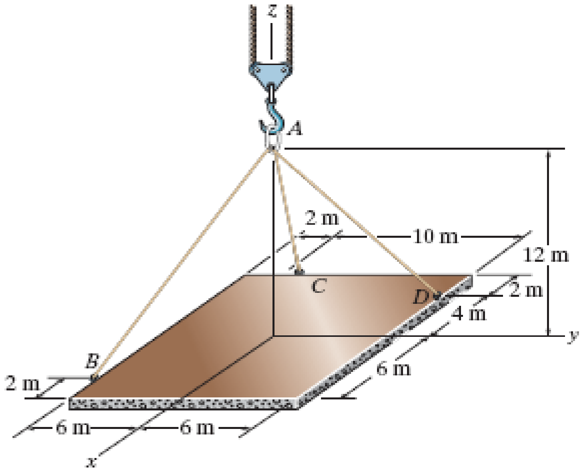
$F_{AB} = 2.52$ kN **Ans**

$F_{CB} = 2.52$ kN **Ans**

$F_{BD} = 3.64$ kN **Ans**



3–56. The ends of the three cables are attached to a ring at *A* and to the edge of a uniform 150-kg plate. Determine the tension in each of the cables for equilibrium.

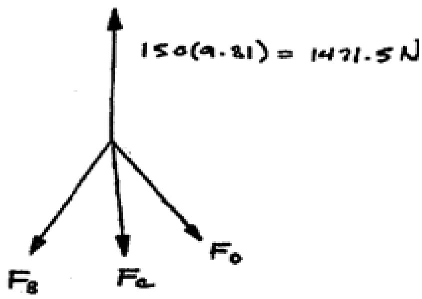


$$\mathbf{P} = 150(9.81)\mathbf{k} = 1471.5\,\mathbf{k}$$

$$\mathbf{F}_B = \frac{4}{14}F_B\,\mathbf{i} - \frac{6}{14}F_B\,\mathbf{j} - \frac{12}{14}F_B\,\mathbf{k}$$

$$\mathbf{F}_C = -\frac{6}{14}F_C\,\mathbf{i} - \frac{4}{14}F_C\,\mathbf{j} - \frac{12}{14}F_C\,\mathbf{k}$$

$$\mathbf{F}_D = -\frac{4}{14}F_D\,\mathbf{i} + \frac{6}{14}F_D\,\mathbf{j} - \frac{12}{14}F_D\,\mathbf{k}$$



$$\Sigma F_x = 0; \qquad \frac{4}{14}F_B - \frac{6}{14}F_C - \frac{4}{14}F_D = 0$$

$$\Sigma F_y = 0; \qquad -\frac{6}{14}F_B - \frac{4}{14}F_C + \frac{6}{14}F_D = 0$$

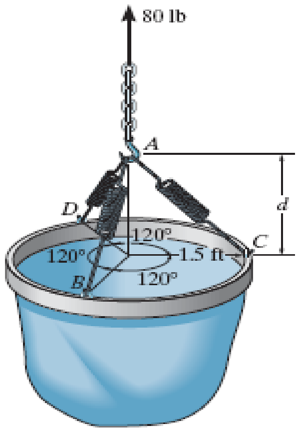
$$\Sigma F_z = 0; \qquad -\frac{12}{14}F_B - \frac{12}{14}F_C - \frac{12}{14}F_D + 1471.5 = 0$$

$F_B = 858$ N **Ans**

$F_C = 0$ **Ans**

$F_D = 858$ N **Ans**

3–66. The bucket has a weight of 80 lb and is being hoisted using three springs, each having an un-stretched length of lo = 1.5 ft and stiffness of k = 50 lb/ft . Determine the vertical distance *d* from the rim to point *A* for equilibrium.



$\Sigma F_z = 0;$

$$80 - \left(\frac{3d}{\sqrt{d^2 + (1.5)^2}}\right) F = 0$$

$$80 - \frac{3d}{\sqrt{d^2 + (1.5)^2}}\left[50\left(\sqrt{d^2 + (1.5)^2} - 1.5\right)\right] = 0$$

$$\frac{d}{\sqrt{d^2 + (1.5)^2}}\left(\sqrt{d^2 + (1.5)^2} - 1.5\right) = 0.5333$$

$$d\sqrt{d^2 + (1.5)^2} - 1.5\,d = 0.5333\sqrt{d^2 + (1.5)^2}$$

$$\sqrt{d^2 + (1.5)^2}\,(d - 0.5333) = 1.5\,d$$

$$\left[d^2 + (1.5)^2\right]\left[d^2 - 2d\,(0.5333) + (0.5333)^2\right] = (1.5)^2 d^2$$

$$d^4 - 1.067\,d^3 + 0.284\,d^2 - 2.4\,d + 0.64 = 0$$

$$d = 1.64 \text{ ft} \qquad \textbf{Ans}$$